Nondegenerate Abnormality and Gap Phenomena in Optimal Control with State Constraints

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ABSTRACT

In optimal control theory, *infimum gap* means that there is a gap between the infimum value of a given optimization problem and an auxiliary problem, obtained by first extending the set of original controls and then convexifying the extended velocities set. In this talk we present sufficient conditions for the absence of an infimum gap for a wide class of optimal control problems subject to endpoint and state constraints. These conditions, which also guarantee controllability of the original system to an extended solution, are based on a nondegenerate version of the nonsmooth constrained Maximum Principle, expressed in terms of subdifferentials. In particular, under some new constraint qualification conditions based on [1, 2] we prove the following theorem.

Theorem. If a (relaxed) extended minimizer is a *nondegenerate normal* extremal, i.e. every set of nondegenerate multipliers has cost-multiplier greater than zero, then there is no infimum gap.

This result enhances the outcomes of the previous literature. In fact, in order to prove that there is no infimum gap, it is sufficient to show that normality holds not for all sets of multipliers (see for instance [5, 6, 7]), but only among the nondegenerate ones. As a consequence, we are able to include the common case of fixed initial state with active state constraint, situation in which every extremal is abnormal. A final example will capture this circumstance, so that to illustrate our theorem and the improvements with respect to the previous literature mentioned above. See also [3, 4].

This is a joint work with *Monica Motta*, from *University of Padova*.

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