Unbounded Control, Infimum Gaps, and Higher Order Normality

Monica Motta

Department of Mathematics "Tullio Levi-Civita", University of Padua, Italy

ABSTRACT

In optimal control theory one sometimes extends the minimization domain of a given problem, with the aim of achieving the existence of an optimal control. However, this issue is naturally confronted with the possibility of a gap between the original infimum value and the extended one. Avoiding this phenomenon is not a trivial issue, especially when the trajectories are subject to endpoint constraints. Since the seminal works by J. Warga in 1970s [3, 4], some authors have recognized 'normality' of an extended minimizer as a condition guaranteeing the absence of an infimum gap. (Let us recall that an extremal is called *abnormal* provided the corresponding cost multiplier in the Maximum Principle can be chosen equal to zero, and *normal* otherwise.) In particular, in [2] a generalization of Warga's criterion to a vast class of endpoint-constrained minimum problems' extensions has been recently achieved through the combined use of the notion of *abundance* (see [1] and [5, 6]) and of a suitable set separation theorem.

Yet, normality is far from being necessary for this goal, a fact that makes the search for weaker assumptions a reasonable aim. In relation with a control-affine system with unbounded controls, we provide a sufficient no-gap condition based on a notion of *higher order normality*, which is less demanding than the standard normality and involves iterated Lie brackets of the vector fields defining the dynamics.

This is a joint work with *Michele Palladino* from *Gran Sasso Science Institute - GSSI, L'Aquila, Italy Franco Rampazzo* from *Department of Mathematics "Tullio Levi-Civita", Padua University, Italy.*

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