State constraints, Lie brackets, and higher order inward pointing conditions

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ABSTRACT

Under the standard *inward pointing condition* (IPC) for a control system $\dot{x} = \sum_{i=1}^{m} g_(x)u^i$ whose state x is constrained in the closure $\bar{\Omega}$ of an open set $\Omega \subset \mathbf{R}^n$, every system trajectory $x : [0,T] \to \mathbf{R}^n$, $x(0) \in \bar{\Omega}$, possibly violating the constraint, can be approximated by a new system trajectory $\hat{x}(\cdot)$ that satisfies the constraint and whose distance from $x(\cdot)$ is bounded by a quantity proportional to $d := \sup\{dist(\Omega, x(t)),\ t \in [0,T]\}$. To treat the case when (IPC) is violated, in [1] one assumes a *higher order inward pointing condition involving the Lie brackets* $[g_i, g_j]$, $i,j = 1,\ldots,m$, together with a *non-positive-curvature-like assumption.* Under these hypotheses, the implementation of a suitable rotating control strategy allows for a novel construction of a constrained trajectory $\tilde{x}(\cdot)$ whose distance from $x(\cdot)$ is bounded by a quantity proportional to $\sqrt{d}$. As an application, one establishes the continuity of the value function $V : \mathbf{R}^n \backslash \bar{\Omega} \to \mathbf{R}$ of any connected optimal control problem. In particular, this continuity property is sufficient for V to be the *unique* constrained viscosity solution of the corresponding Bellman equation. (As an object for future investigation, one might also envisage the sufficiency of the presented higher order condition for the non-degeneracy of the connected maximum principle.)

References

[1] G.Colombo, N. Khalil, and F.Rampazzo: *State constraints, higher order inward pointing conditions, and neighboring feasible trajectories,* To appear in SIAM Journal on Control and Optimization.