Control systems with paraboloid nonholonomic constraints

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ABSTRACT

In this talk, we present a characterisation and a classification of control systems (both control-affine and fully nonlinear) whose field of admissible velocities forms a paraboloid submanifold in each tangent space. We thus continue previous works on affine and linear Pfaffian equations whose admissible velocities form either an affine or a linear subspace in each tangent space; see [4, 5, 1, 6]. Precisely, we consider a smooth n-dimensional manifold $\mathcal{X}$ equipped with local coordinates x and we call a nonholonomic constraint *paraboloid* if it is given by an equation of the form

$$S_Q(x, \dot{x}) = \dot{z} - \dot{y}^t Q(x) \dot{y} - b(x)\dot{y} - c(x) = 0,$$

where $x = (z, y)$, with $y = (y_1, \ldots, y_{n-1})$, Q is a smooth symmetric matrix of full rank, b is a smooth covector, and c is a smooth function. The case $n = 2$ corresponds to S_Q describing a parabola $\dot{z} = a(x)\dot{y}^2 + b(x)\dot{y} + c(x)$ in each fiber $T_x\mathcal{X}$ and is treated (and generalised to elliptic and hyperbolic cases) in [3] and is extended to all dimensions by the speaker in his thesis [2].

We will introduce a novel class of control-affine systems whose trajectories satisfy the paraboloid nonholonmic constraint given by S_Q and we give a characterisation of that class among all control-affine systems. Next, we will classify paraboloid control systems under feedback transformations. In particular, we will insist on the differences between the cases $n = 2$ and $n \geq 3$. Moreover, we will show how our classification of control systems provides a corresponding classification of paraboloid equations $S_Q = 0$.

This is a joint work with *Witold Respondek* from *INSA Rouen Normandie.*

References

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